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Medical Waste Collection Problem: A Case Study in Eskişehir

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Abstract

Medical waste generated by hospitals and other medical service points need to be properly disposed of by municipalities in large metropolitan areas. To this end, special vehicles and containers are used in the collection of such waste. The location of the medical service points is known in advance. It is also assumed that the amounts of waste at the medical service points are deterministic and known. Each vehicle starts its route from a central location called the depot and visits a subset of medical service points collecting the waste and delivers to a common incineration/sterilization station. All vehicles are expected to return to the depot after delivery. The objective of this study is to develop a mathematical model to minimize the total distance travelled by all vehicles. The medical waste collection problem is formulated as a classical capacitated Vehicle Routing Problem. A small sized instance of the problem is solved to optimality using the GAMS software. It also generates optimal routes for each vehicle. The results are compared to the current practice and found to result in important reductions in total distance traveled.

Keywords: Medical waste management, Optimization, Vehicle routing problem

1. INTRODUCTION

Collection and disposal of medical waste is one of the critical tasks that many municipalities face in all over the world. Medical waste is generated at different locations in a city such as hospitals, clinics, laboratories, blood banks and generally a local private or municipal authority is responsible for the collection, transport and proper disposal of medical waste.

As the accumulation of medical waste at the medical service points is undesired, regular collection of such waste is critical especially when infectious medical waste is in question.

Generally, the task of collecting medical waste from each source points is the subject of a local business which seeks to minimize its transportation costs incurred by the fleet of special vehicles. Thus from an optimization point of view, the problems becomes to identify a closed route starting and ending at the depot for each vehicle so as to minimize the total distance travelled by all vehicles. This optimization problem is known as the Vehicle Routing Problem (VRP) which belongs to one of the hardest problem class (NP-Complete).[1]

There are a number of extensions of the VRP but four of them are especially relevant to medical waste collection:

i) the capacitated vehicle routing problem (CVRP) where each vehicle has a predetermined capacity [2],

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ii) the load-dependent vehicle routing problem where the transportation cost depends on the load actually present on the vehicle [3],

iii) Periodic vehicle routing problem (PVRP) where the frequency of visit may be different at different routes [4],

iv) Green inventory problem where the cost of pollution from the vehicles is considered [5].

In this study, although the Periodic VRP might be more relevant to the problem on hand, we considered daily collection of medical waste leading to a simpler version of the problem namely, CVRP.

2. MATERIALS AND METHODS

The following mixed integer linear programming model is adopted from [6].

2.1. Model and Assumptions

In this model we assume the following:

* Each vehicle starts from and returns to a single location called depot.

* Vehicles are identical except for the capacity.

* Each medical center is visited by a single vehicle.

* The unit cost of collection is directly proportion to the distance travelled by the vehicle, thus the minimization of distance travelled by all vehicles imply the minimization of total cost. We disregard all other costs associated with the collection of medical waste.

Let n denote the number of medical centers and $C = \{1, 2, 3, \ldots, n\}$ represent the set of medical centers. Also let 0 denote the incineration/sterilization station (depot) from which all vehicles start from and return to. Consider the complete and directed graph $G = (N, E)$ represent the vehicle routing network where N is the set of all locations including the depot $N = \{0, 1, 2, 3, \ldots, n\}$ and $E = \{(i, j) : i, j \in N, i \neq j\}$ is the set of all arcs of the graph G.

Let m denote the number of vehicles and the set $K = \{1, 2, \ldots, m\}$ represents the set of vehicles. Other parameters of the model are given below:

Cap_k : the capacity of the vehicle $k \in K$ (in kg.)

d_{ij} : the distance between nodes $i \in N$ and $j \in N$

q_i : the quantity of medical waste to be collected from medical center $i \in C$

The distances are measured on the Google map between considering the shortest paths. We also assume a symmetric distance matrix.

Decision Variables:

$$y_{ik} = \begin{cases} 1 & if\ center\ i\ is\ visited\ by\ vehicle\ k \\ 0 & otherwise \end{cases}$$

$$x_{ijk} = \begin{cases} 1 & if\ center\ j\ is\ visited\ just\ after\ i\ \ by\ the\ vehicle\ k \\ 0 & otherwise \end{cases}$$

$u_{ik} =$ the actual load on the vehicle $k \in K$ after visiting center $i \in C$

The mathematical model can be stated as follows:

$$Min\ \ z = \sum_{i \in N} \sum_{j \in N} d_{ij} \sum_{k \in K} x_{ijk} \tag{1}$$

Subject to:

$$\sum_{i \in N} q_i \ y_{ik} \leq Cap(k) \qquad k \in K \tag{2}$$

$$\sum_{k \in K} y_{ik} = 1 \qquad i \in C \tag{3}$$

$$\sum_{k \in K} y_{0k} = m \tag{4}$$

$$\sum_{j \in N} x_{ijk} = \sum_{j \in N} x_{jik} = y_{ik} \qquad i \in N, k \in K \tag{5}$$

$$u_{ik} - u_{jk} + Cap(k) * x_{ijk} \leq Cap(k) - q_i \quad i,j \in C, i \neq j \ such \ that \ q_i + q_j \leq Cap(k), k \in K \tag{6}$$

$$q_i \leq u_{ik} \leq Cap(k) \qquad i \in C, k \in K \tag{7}$$

$$x_{ijk} \in \{0,1\} \qquad i,j \in N, k \in K \tag{8}$$

$$y_{ik} \in \{0,1\} \qquad i \in N, k \in K \tag{9}$$

The objective function (1) minimizes the total distance travelled by all vehicles. The constraints (2) are vehicle capacity constraints and ensure that the capacity of each vehicle is not exceeded. Constraint (3) makes sure that each medical center is visited by exactly one vehicle. Here, we assume that there exists a vehicle whose capacity is sufficiently large to handle the load by itself for each medical center. Constraint (4) ensures that all vehicles leave the depot. Constraints (5) connect the decision variables x and y making a continuous path for the vehicles. The constraints (6) and (7) are subtour elimination constraints. This version of constraints is first proposed by Meller-Tucker-Zemlin [7]. Constraints (8) and (9) are binary constraints.

2.2. Case Study

In this case study, we consider the medical waste collection problem in the Eskişehir area. Although there are a number of medical centers in the area, we only consider the largest 10 private and public medical institutions in the area. The company own two vehicles especially designed for the collection and transportation of medical waste. Capacity information is given in Table 1. Normally, the collection pattern or period should differ from center to center: the higher the quantity the more frequent the visits should be. In this case, during the latest Covid-19 pandemic daily visits to centers are expected and we schedule a single visit to each center daily. Thus, the theoretical problem becomes a classical capacitated VRP as opposed to a Periodic VRP. Distances taken by the vehicles are directly measured from Google Maps web site and distance values represent the shortest actual travel distances on the existing road network. The distances are given in Table 2. The quantity of waste to be collected for each medical center is difficult to find directly, they are estimated via the total number of inpatient beds including the intensive care units. Also this number is increased during the Covid-19 pandemic period and we considered three different scenarios with increased waste quantities.

Table 1. Capacities of the collection vehicles

Vehicle Number	Capacity (kg)
1	2000
2	1500

Table 2. Travel distances between locations (km)

	0	1	2	3	4	5	6	7	8	9	10
0	0.0	10.7	13.1	13.4	13.5	7.0	12.7	8.2	8.0	52.2	70.7
1	10.7	0.0	12.1	7.6	14.7	11.2	12.7	9.3	8.8	52.3	60
2	13.1	12.1	0.0	5.2	2.6	7.4	1.8	7.2	4.3	41.2	72.1
3	13.4	7.6	5.2	0.0	7.2	10.6	5.2	10.4	4.5	45.2	67.6
4	13.5	14.7	2.6	7.2	0.0	7.4	3.1	8.3	6.8	38.9	74.1
5	7.0	11.2	7.4	10.6	7.4	0.0	5.8	1.7	3.9	45.4	71.1
6	12.7	12.7	1.8	5.2	3.1	5.8	0.0	7.6	2.9	41.1	71.7
7	8.2	9.3	7.2	10.4	8.3	1.7	7.6	0.0	2.2	46.3	69.5
8	8.0	8.8	4.3	4.5	6.8	3.9	2.9	2.2	0.0	43.8	70.8
9	52.2	52.3	41.2	45.2	38.9	45.4	41.1	46.3	43.8	0	112
10	70.7	60	72.1	67.6	74.1	71.1	71.7	69.5	70.8	112	0

3. *RESULTS AND DISCUSSION*

The model is coded in GAMS using CPLEX solver and optimal solutions are obtained within 5 minutes given the small size of the input data. The optimal vehicle routes are given in Table 3 and a Google view is shown in Figure 1.

Table 3. Optimal vehicle routes with k=2 vehicles

Vehicle #	Vehicle Routes	Total distance travelled (km)	Total load of the route (kg)	% capacity used
1	0 - 7 - 5 - 6 - 2 - 4 - 0	33.6	2000	100
2	0 - 8 - 9 - 3 -10 - 1 0	235.3	1500	100
		268.9	**3500**	

According to the optimal solution, vehicle #1 visits centers 7, 5, 6, 2 and 4 before returning to the depot. A distance of 33.6 km is taken by the vehicle and 100% of the capacity is used.

Similarly, vehicle #2 visits centers 8, 9, 3, 10 and 1 before returning to the depot. A distance of 235.3 km is taken by the vehicle and 100% of the capacity is used. It is a surprising coincidence that both vehicles are used at the 100% capacity level.

Another surprising fact is that there is a large difference in the distance travelled by the vehicles. This is mainly due to the fact that two of the medical centers are located outside city center are visited by the same vehicle (#2) while remaining centers which are located relatively close to each other around the city center are visited by the other vehicle (#1).

In the next section, we introduce a distance constraint to avoid this undesirable situation.

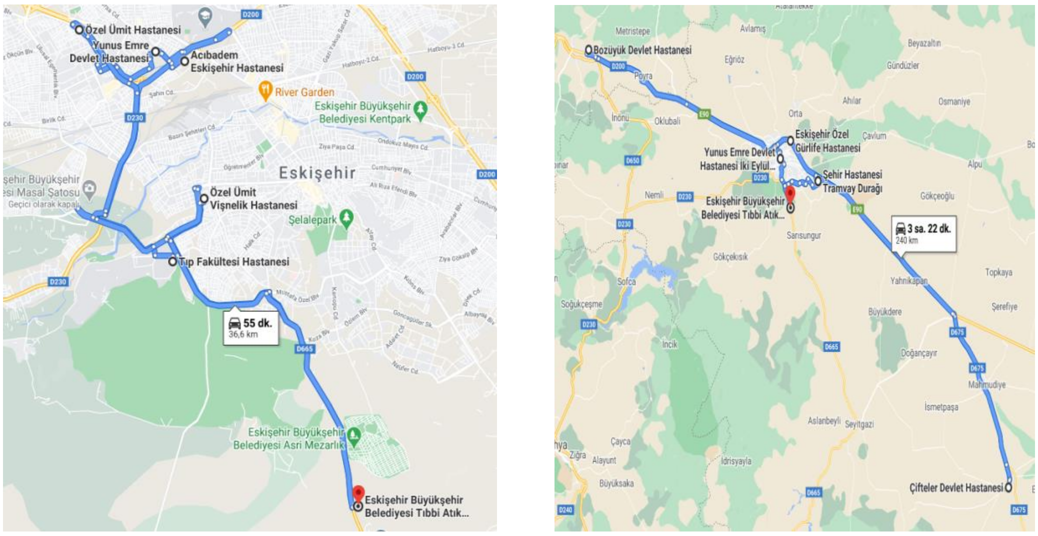

Figure 1. Google view of the optimal routes for vehicle #1 on the left and #2 on the right

3.1. Distance Constraints

On the inspection of the optimal routes given Table 3, we observe that a large gap between the distances travelled by the vehicles. We impose the following distance constraint (10) into the model:

$$\sum_{i \in N} \sum_{j \in N} d_{ij} x_{ijk} \leq Distance\ limit\ (k) \tag{10}$$

Constraint (*) makes sure that the distance taken by vehicle k, must not exceed a distance limit denoted by the parameter "*Distance limit (k)*". A 150 km. distance limit is considered for each vehicle.

In our case since both vehicles are fully loaded at 100% capacity it is not possible to reduce the travel distances without adding an additional vehicle. Therefore, we introduce a third vehicle with capacity 1500 kg.

Table 4. Optimal vehicle routes with k=3 vehicles for base case scenario

Vehicle #	**Optimal route**	**Total distance travelled (km)**	**Total load of the route (kg)**	**% capacity used**
1	0 - 5 - 0	14.0	1010	51
2	0 -1 - 10 - 0	141.4	1121	75
3	0 - 3 - 2 - 4 - 9 - 6 - 8 - 7 - 0	114.5	1369	91
		269.9	**3500**	

Under the Covid-19 pandemic, the quantity of medical waste collected is expected to increase. Let us call the current level of waste collection "base case scenario", and we generate three more cases by increasing the quantity of collected waste by 25%, 50% and 75% on top of the base case scenario. The optimal number of vehicles and routes are shown in Tables 5-7. In the following scenarios all vehicles are assumed to have a capacity of 2000 kg. and no distance limit.

Table 5. Optimal vehicle routes with k=3 vehicles (base +25%)

K=3	**Vehicle #**	**Vehicle routes**	**Total distance travelled (km)**	**Total load of the route (kg)**	**% capacity used**
25%	1	0 - 3 - 2 - 4 - 9 - 6 - 8 - 7 - 0	114.5	1711.25	86
	2	0 - 5 - 0	14.0	1262.50	63
	3	0 - 1 - 10 - 0	141.4	1401.25	70
			269.9	**4375.00**	

Table 6. Optimal vehicle routes with k=3 vehicles (base +50%)

K=3	**Vehicle #**	**Vehicle routes**	**Total distance travelled (km)**	**Total load of the route (kg)**	**% capacity used**
50%	1	0 - 5 - 7 - 8 - 0	18.9	1672.50	84
	2	0 - 6 - 9 - 4 - 2 - 3 - 0	113.9	1896.00	95
	3	0 - 1 - 10 - 0	141.4	1681.50	84
			274.2	**5250.00**	

Table 7. Optimal vehicle routes with k=4 vehicles (base +75%)

K=4	**Vehicle #**	**Vehicle routes**	**Total distance travelled (km)**	**Total load of the route (kg)**	**% capacity used**
75%	1	0 - 5 - 0	14.00	1767.50	88
	2	0 - 10 - 1 - 0	141.40	1961.75	98
	3	0 - 8 - 3 - 0	25.90	400.75	20
	4	0 - 7 - 9 - 4 - 2 - 6 - 0	110.50	1995.00	100
			291.80	**6125.00**	

It can be observed that as the quantity of waste is increased from base level to 75%, the number of vehicles required increases from 2 to 4, and the total distance traveled by the vehicles increases gradually from 268.9 km to 291.8 km.

In the current practice, medical waste is collected at more than 50 locations. For a fairer comparison, the points not covered by this case study are removed and the resulting simplified routes are shown in Table 8.

Table 8. Vehicle routes (simplified)

Vehicle #	Route	Total distance travelled (km)	Total load of the route (kg)
1	0 -1 - 10 - 0	141.4	1121
2	0 - 3 - 2 - 4 - 9 - 6 - 0	113.9	1264
3	0 - 5 - 7 - 8 - 0	18.9	1115
		274.2	**3500**

4. CONCLUSION

In this case study, the medical waste collection problem in Eskisehir city center and nearby areas is considered. Optimal routes are found for collection vehicles considering only the ten largest medical institutions. Moreover, a projected 25%, 50% and 75% increase in collected waste is investigated Comparing Tables 4 and 8, a saving of 4.3 km per day is achieved. The amount of saving becomes much higher when all medical centers are considered for a year.

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